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USING PERCENTAGE TO DETERMINE THE VALIDITY OF INTERMEDIATE SYLLOGISMS

Master of Science thesis

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Examiner and topic approved by the Faculty Council of the Faculty of Computing and Electrical Engineering on 9th December 2015

ABSTRACT

LIANGBIN SHEN: USING PERCENTAGE TO DETERMINE THE VALIDITY OF INTERMEDIATE SYLLOGISMS
Tampere University of Technology
Master of Science thesis, 50 pages, 0 Appendix pages
Dec 2015
Master's Degree Programme in Information Technology
Major: Mathematics
Examiner: Prof. Esko Turunen
Keywords: Logic, Aristotelian syllogisms, Percentage method, Intermediate syllogisms

Syllogism is a branch of logic which is related to mathematics and philosophy very much. A syllogism is a kind of logical argument which has two or more propositions that are asserted to be true and a conclusion is given based on the propositions. Aristotelian syllogisms are well known as they are the oldest and most classical syllogism. They include three parts: the major premise, the miner premise, and the conclusion. With the development of the study of mathematics, 24 valid ones of Aristotelian syllogisms have been found.

Intermediate syllogisms are extensions of Aristotelian syllogisms. They also include the same three parts. However, there are more than two quantifiers while the Aristotle syllogisms have two quantifiers "All" and "Some". Peterson's Intermediate syllogisms have five quantifiers and many studies about this kind of syllogism have been done by L. Peterson, Turunen and etc. This thesis will first talk about the work done by others and then a new terminology will be introduced to determine the validity of Intermediate syllogisms. At last, some examples will be given for testing.

PREFACE

This Master of Science Thesis has been carried out at the Department of Mathematics in Tampere University of Technology (TUT) in 2015.

Firstly, I would like to thank my supervisor Professor Esko Turunen, because he gave me such an interesting topic. And thanks to his help, I can do the job independently and successfully.

Secondly, I would like to thank people I know and people I do not know here, they are friendly and patient.

Finally, I would like to thank my parents, their support is unconditional and eternal. I love you, my father and my mother!

Tampere, 10.12.2015

Liangbin Shen

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1. INTRODUCTION

1.1 Motivation

A syllogism is a kind of logical argument which has two or more propositions that are asserted to be true and a conclusion is given based on the propositions. Aristotelian syllogisms are well known as they are the oldest and most classical syllogisms. They include three parts: the major premise, the miner premise, and the conclusion. For example, if we have "all human should drink water" as the major premise, and "Mary is a human" as the miner premise, then we can easily find the conclusion that "Mary should drink water". The example can be showed as the following:

$$\frac{\begin{array}{c}\text{All human should drink water}\\ \text{Mary is a human}\end{array}}{\text{Mary should drink water}}$$

What's more, there are only two quantifiers "All" and "Some" in the Aristotelian syllogisms. For example

$$\frac{\begin{array}{c}\text{if "All M are P"}\\ \text{and "Some S are M"}\end{array}}{\text{then "Some S are P"}}$$

is such a syllogism called *Darii* since ancient times. There are 256 possible Aristotelian syllogisms while only 24 of them are valid ones [2]. We will show what is the 24 valid ones later in the thesis. We will also explain why they are valid or not and give examples.

Intermediate syllogisms are extensions of Aristotelian syllogisms. They also include the same three parts. However, there are more than two quantifiers while the Aristotle syllogisms have two quantifiers "All" and "Some". Peterson's Intermediate syllogisms have five quantifiers "All", "Almost-all", "Most", "Many" and "Some" [1]. For example

if "Many M are P"
<u>and "All M are S"</u>
then "Some S are P"

is such an Intermediate syllogism.

And in Peterson's book(Peterson 2000), he used detailed linguistic method and Venn diagram to prove that there are 105 valid syllogisms while 3895 syllogisms are not valid. Peterson also gives the 105 valid syllogisms clearly. However, Turnen found that some results in Peterson's book may be wrong when he studied the Intermediate syllogism based on the MV-algebra in 2014.

Except the different results above, another question should be addressed. The question is that how many valid syllogisms there will be when we introduce new quantifiers. For example, based on the Peterson's Intermediate syllogisms, another quantifier "Half" are added and there will be six quantifiers. It is very important for us to find a beautiful method to solve such problem. Therefore, this thesis work is mainly related to intermediate syllogisms not only have five but could have many quantifiers.

The purpose of the thesis is to introduce a new terminology called percentage method to determine the validity of Peterson's Intermediate syllogisms. And we will also prove this method for syllogisms have more than five quantifiers. At last, we will give examples and compare this method with others' work.

1.2 Thesis organization

There are six chapters in this thesis, details are as follow:

Chapter 2: introduce the concept of syllogisms, and explores the valid ones of Aristotelian syllogisms. Based on the syllogisms, Peterson's Intermediate syllogisms are introduced. Others' work on the syllogisms and both advantages and disadvantages will be talked about.

Chapter 3: introduces the percentage method. And we will use the Venn diagram and logic reasoning to give several theorems to prove the method. This method will not be only used in five quantifiers but also more than five quantifiers.

Chapter 4: In this chapter, we will give both theoretical example and practical example.

Chapter 5: The percentage method will be compared with others' work. Advantages and disadvantages will be listed.

Chapter 6: The results of the percentage method will be showed and future research directions will be presented.

2. HISTORY AND BACKGROUND

2.1 Aristotelian syllogisms

2.1.1 Logic

Aristotle defined mathematics as "the science of quality" in ancient times [7]. Logic is the branch of philosophy concerned with the use and study of valid reasoning and related to mathematics very much especially in nowadays [4, 5]. Modern formal logic follows and expands on Aristotle [6].

The study of logic was begun by the ancient Greeks whose educational system stressed competence in reasoning and in the use of language. Along with rhetoric and grammar, logic formed part of the trivium, the first subjects taught to young people. Rules of logic were classified and named [3]. The most widely known set of rules are the syllogisms which we will talk later.

2.1.2 Aristotelian syllogisms

In antiquity, there are two kinds of syllogism: Aristotelian syllogism and Stoic syllogism [8]. Aristotle defines the syllogism as a discourse in which certain (specific) things having been supposed, something different from the things supposed results of necessity because these things are so [9]. In Aristotle's work, he limits himself to categorical syllogisms that consist of three categorical propositions. These include categorical modal syllogisms.

From ancient time to nowadays, many work about Aristotelian syllogisms have been done until the arise of modern mathematical logic [2]. However, as a kind of basic theory of logic, the scientific research of Aristotelian syllogism is still taking place especially in the field of proofing validity of syllogisms. It is known that there exists a sound and complete proof system for the classical syllogistic in the form of a finite set of syllogism-like proof-rules [17].

There are three parts in Aristotelian syllogisms:

1. Major premise;

2. Miner premise;

3. Conclusion.

Aristotelian syllogisms are always presented in a three-line form. Every line is a statement. For example:

$$\frac{\begin{array}{c}\text{All natural numbers are rational numbers (Major premise)}\\ \text{All rational numbers are real numbers (Miner premise)}\end{array}}{\text{All natural numbers are real numbers (Conclusion)}}$$

is a simple Aristotelian syllogism. Based on the reasoning logic, we can easily conclude that "All natural numbers are real numbers" from the first two premise "All natural numbers are rational numbers" and "All natural numbers are real numbers".

As we can see from the example above, each part is a statement which contains two terms. In the conclusion there are two parts: a *subject* S and a *predicate* P. And in the two premise, there is a *middle term* M in every statement. So it is easy to know the relationship between S and P (Conclusion) if we know the relation between M and P (Major premise) and the relation between S and M (Miner premise). In this example, "natural numbers" are S, "real numbers" are P and "rational numbers" are M.

And as we already show that Aristotelian syllogisms have two quantifiers: "All" and "Some", there are four kinds of statements for each part. The four kinds of statements are "All x are y", "All x are not y", "Some x are y" and "Some x are not y", where "x" and "y" are terms. The statements with "All" are called *universal* propositions while the other two with "Some" are called *particular* propositions. We also could divided them into affirmative statements and negative statements. Here is the table for the four types:

Table 2.1: *Four types of statements in syllogisms*

code	statement	type
A	All x are y	universal affirmatives
E	All x are not y	universal negatives
I	Some x are y	particular affirmatives
O	Some x are not y	particular negatives

In the conclusion, the position of S and P is unchangeable, While the positions of terms in the premise can be changed. So based on this situation, the Aristotelian syllogisms can be classified as what we called *Figures.* There are four *Figures* as the following:

Figure I

$$\frac{\begin{array}{c}\text{A quantity } Q_1 \text{ of M are P (Short as MP)}\\ \text{A quantity } Q_2 \text{ of S are M (Short as SM)}\end{array}}{\text{A quantity } Q_3 \text{ of S are P}}$$

Figure II

$$\frac{\begin{array}{c}\text{A quantity } Q_1 \text{ of P are M (Short as PM)}\\ \text{A quantity } Q_2 \text{ of S are M (Short as SM)}\end{array}}{\text{A quantity } Q_3 \text{ of S are P}}$$

Figure III

$$\frac{\begin{array}{c}\text{A quantity } Q_1 \text{ of M are P (Short as MP)}\\ \text{A quantity } Q_2 \text{ of M are S (Short as MS)}\end{array}}{\text{A quantity } Q_3 \text{ of S are P}}$$

Figure IV

$$\frac{\begin{array}{c}\text{A quantity } Q_1 \text{ of P are M (Short as PM)}\\ \text{A quantity } Q_2 \text{ of M are S (Short as MS)}\end{array}}{\text{A quantity } Q_3 \text{ of S are P}}$$

We can see that there are three statements: Major premise, Miner premise and Conclusion. And every statement has four types while there are four *Figures*, so there are 4 * 4 * 4 * 4 = 256 possible Aristotelian syllogisms. We can use different abbreviation to represent different syllogisms. For example, AAI-III means the syllogism is in the form of *Figures* III, and the first two premise are universal affirmatives while the conclusion is particular affirmatives. Simply, it can be presented as the following three-line form:

$$\frac{\begin{array}{c}\text{All M are P}\\ \text{All M are S}\end{array}}{\text{Some S are P}}$$

For another example, EIO-II can be presented as:

$$\frac{\begin{array}{c}\text{All P are not M}\\ \text{Some S are M}\end{array}}{\text{Some S are not P}}$$

For this technique, it is very simple to present the syllogisms in a clear way.

2.1.3 Valid syllogisms

A syllogism is *valid* if, whenever some relation between M and S is assumed to hold and another relation between M and P is assumed to hold, then we can conclude that certain relation between S and P would hold [2]. For example,

$$\frac{\begin{array}{c}\text{All rabbits have fur}\\ \text{Some pets are rabbits}\end{array}}{\text{Some pets have fur}}$$

is a valid syllogism because we can easily conclude the conclusion. However,

$$\frac{\begin{array}{c}\text{All rabbits have fur}\\ \text{Some pets are rabbits}\end{array}}{\text{All pets have fur}}$$

is invalid as some pets like fish do not have fur. We can not conclude the conclusion "All pets have fur" from the premise.

Table 2.2: *Valid Aristotelian syllogisms for every figure*

Figure I	Figure II	Figure III	Figure IV
AAA	AEE	AAI	AAI
AAI	AEO	AII	IAI
AII	AOO	IAI	AEE
EAE	EAE	EAO	AEO
EAO	EAO	EIO	EAO
EIO	EIO	OAO	EIO

There are 256 possible Aristotelian syllogisms but only 24 of them are valid. The Table 2.2 is the 24 valid Aristotelian syllogisms.

Considering the validity of every syllogism would cost a lot of time. Luckily, a lot of people have find many simple ways to solve the problem. During these solution, Venn diagrams are more directly to the validity of syllogisms [10].

In the Venn diagram Figure 2.1, we view the subject S, the predictor P and the middle term M as three sets. And the natural numbers a, b, c, d, e, f and g refer to the number of elements in the corresponding area. S, P and M are assumed to be non-empty which means if all S are M then a + b = 0 while d + e > 0. This is called *Principle of imported existence* [2], we never consider empty sets.

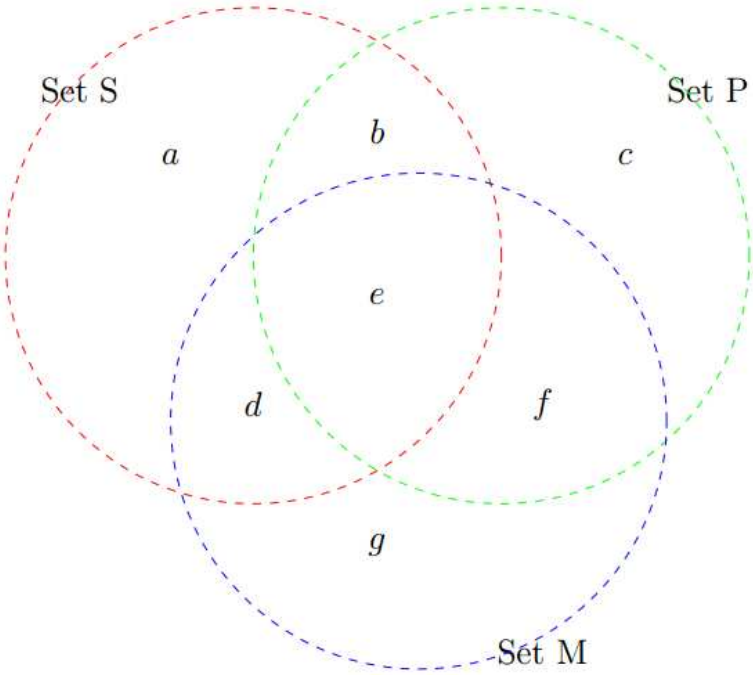

Figure 2.1: *Venn diagram for Aristotelian syllogisms*

Let's give two examples to show how can we use the Venn diagram to prove the validity of Aristotelian syllogisms.

Example 1. EIO-IV. It is a valid syllogism and can be presented as:

All P are not M
<u>Some M are S</u>
Some S are not P

From the major premise "All P are not M" which means "No P is M", we can conclude that e + f = 0 and b + c > 0; "Some M are S" means d + e > 0; Because we already have e = f = 0 in the Major premise, it is necessary that d > 0 as d + e > 0. This proves there is some S (at least d>0 elements) are not P.

Example 2. AEO-I.

All M are P
<u>All S are not M</u>
Some S are not P

Here, we can see that d + g = 0 and e + f > 0 from the Major premise; and d + e = 0 , a + b > 0 from the Miner premise. We can conclude d = e = 0, however, whether a = 0 is unknown as we can assigned a = 0 and b > 0 to satisfy the *Principle of imported existence*. Then in this case, some S are P. So the Aristotelian syllogism AEO-I is invalid.

We have talked about the definition of the syllogisms and the validity of Aristotelian syllogisms in this section. Also, we give examples to show how to use Venn diagram to determine the validity of Aristotelian syllogisms.

2.2 Intermediate syllogisms

2.2.1 Intermediate quantifiers

In linguistics and semantics, a quantifier means the quantity of something. "a lot", "some" and "many" are such quantifiers. We can see that in Aristotelian syllogisms, there are two quantifiers: universal quantifier "All" and particular quantifier "Some".

However, in natural language, only "All" and "Some" is not enough. It is necessary to introduce other quantifiers to satisfy the natural language [13, 14]. Except the "All" and "Some", quantifiers like "Many", "a small part of" and so on are called *generalized quantifiers* or *intermediate quantifiers*.

2.2.2 Peterson's Intermediate syllogisms

We can see that there are only two quantifiers "All" and "Some" in the Aristotelian syllogisms. If a syllogism contains not only the two basic quantifiers "All" and "Some" but also other intermediate quantifiers, then we call it *Intermediate syllogism*. Some research studies and formal theory about the Intermediate syllogisms have already be done [11, 12].

Peterson gives another three intermediate quantifiers "Almost-all", "Most" and "Many" into the syllogisms, so there are five quantifiers in this kind of syllogisms. We call it Peterson's Intermediate syllogisms.

Just the same as the Aristotelian syllogisms, there are four Figures and three statements: Major premise, Miner premise and conclusion. However, as there are five quantifiers, the types of the statements are more complicated. The following is the table of types for Peterson's Intermediate syllogisms:

Table 2.3: *Types of Peterson's Intermediate syllogisms*

Affirmative statements	Negative statements	Generic term
A: All x are y	**E**: All x are not y	universal
P: Almost-all x are y	**B**: Almost-all x are not y	predominant
T: Most x are y	**D**: Most x are not y	majority
K: Many x are y	**G**: Many x are not y	common
I: Some x are y	**O**: Some x are not y	particular

By Peterson's linguistic analysis, "Most x are y" means "More than half of x are y". This is very important for the following analysis.

From the table we can see that statements A and E, P and B, T and D compose contrary *pairs* [1]. They can not be both true at the same time. However, they can be both false. We also say that the *complement* of A, P and T is E, B and D. We call the universal, predominant and majority statements *preponderance statements*. And for statements K and G, I and O, they can be both true of false at the same time, we call them *sub-contrary paris*.

Peterson interpret that if the Statement A "All x are y" is true, than the statement P "Almost-all x are y" is also true. We than call that A is *stronger* than P. P is also stronger than T and T is stronger than K and so on. The same for the negative statements, if E "No x are y" is true, then B, D, G, O are true, E is stronger than the other negative statements. We can also say B is *weaker* than E.

The same as Aristotelian syllogisms, we can also use the form of XYZ-M to represent Peterson's Intermediate syllogism.

For example, PDI-II means the following Intermediate syllogism:

$$\begin{array}{c}\text{Almost-all P are M}\\ \underline{\text{Most S are not M}}\\ \text{Some S are P}\end{array}$$

2.3 Valid Intermediate syllogisms

In this section, we will talk about some researches on Peterson's Intermediate syllogisms and give out valid syllogisms. Also, we will talk about the extension of Peterson's Intermediate syllogisms.

2.3.1 Peterson's work

We can see that there are four Figures as the Aristotelian syllogisms, three statements and every statement could have ten types in Peterson's Intermediate syllogisms, so there are 10 * 10 * 10 * 4 = 4000 Peterson's Intermediate syllogisms. But only a few of them are valid.

As in Aristotelian syllogisms, there is *Principle of imported existence.* In Peterson's Intermediate syllogisms, *Principle of existence import* is also import and it means the sets we talk about are not empty. And based on the study of English quantifiers "Almost-all", "Most" and "Many", Peterson gives several principles related to the valid intermediate syllogisms.

- At least one premise is affirmative;
- The conclusion is negative if and only if one of the premise is negative;
- At least one of the premise must have a quantity of preponderance;
- If any premise is non-universal, then the conclusion must have a quantity that is less than or equal to that premise.

In Peterson's book, he introduces a special Venn diagram method to find the valid intermediate syllogisms. Here are the 105 valid syllogisms in Peterson's approach. The black ones are the Aristotelian syllogisms.

Table 2.4: *Figure I Affirmative*

AAA				
AAP	APP			
AAT	APT	ATT		
AAK	APK	ATK	AKK	
AAI	API	ATI	AKI	**AII**

Table 2.5: *Figure I Negative*

EAE				
EAB	EPB			
EAD	EPD	ETD		
EAG	EPG	ETG	EKG	
EAO	EPO	ETO	EKO	**EIO**

Table 2.6: *Figure II Negative case 1*

AEE				
AEB	ABB			
AED	ABD	ADD		
AEG	ABG	ADG	AGG	
AEO	ABO	ADO	AGO	**AOO**

Table 2.7: *Figure II Negative case 2*

EAE				
EAB	EPB			
EAD	EPD	ETD		
EAG	EPG	ETG	EKG	
EAO	EPO	ETO	EKO	**EIO**

Table 2.8: *Figure III Affirmative*

AAI	PAI	TAI	KAI	**IAI**
API	PPI	TPI	KPI	
ATI	PTI	TTI		
AKI	PKI			
AII				

Table 2.9: *Figure III Negative*

EAO	BAO	DAO	GAO	**OAO**
EPO	BPO	DPO	GPO	
ETO	BTO	DTO		
EKO	BKO			
EIO				

Table 2.10: Figure IV

AAI	**AEE**	**EAO**
PAI	AEB	EPO
TAI	AED	ETO
KAI	AEG	EKO
IAI	**AEO**	**EIO**

As we can conclude from these tables, if the Aristotelian syllogism is valid, then the form of Aristotelian syllogism in Peterson's Intermediate syllogisms is also valid. And the invalid syllogisms in Aristotelian syllogisms will stay invalid in Peterson's Intermediate syllogisms.

And let's consider more, if there are other Intermediate syllogisms which will have six or seven quantifiers based on the Peterson's Intermediate syllogism, the valid ones will also respect the rule above. For example, AKI-III is valid in Peterson's Intermediate syllogisms, so it will remain valid when there is more quantifiers (the extension of Peterson's Intermediate syllogism). ADE-II is invalid in Peterson's Intermediate syllogisms, so it will stay invalid in the extensions.

What's more, we can easily see that the valid Aristotelian syllogisms are in the corner of these tables. And these tables seem to have some triangle rules.

In Peterson's book, it is said that if we add a new quantifier to the five-quantifier Intermediate syllogisms, the valid ones will construct tables like the form in Peterson's Intermediate syllogisms. In Figure I, II, III, it will just add the new quantifier to construct triangle while in Figure IV, it will looks like a rectangle.

For example, if we add a quantifier "Half" which "at least half of them" based on the five-quantifier intermediate syllogisms, then the valid ones in Figure I(Affirmative) should like the following tables:

Table 2.11: Figure I Affirmative + "Half"

AAA					
AAP	APP				
AAT	APT	ATT			
<u>AAF</u>	<u>APF</u>	<u>ATF</u>	<u>AFF</u>		
AAK	APK	ATK	<u>AFK</u>	AKK	
AAI	API	ATI	<u>AFI</u>	AKI	AII

and

Table 2.12: *Figure IV + "Half"*

AAI	AEE	EAO
PAI	AEB	EPO
TAI	AED	ETO
FAI	AEV	EFO
KAI	AEG	EKO
IAI	AEO	EIO

Here, F means "Half or more than x are y" and V means "Half or more than x are not y".

However, Turunen finds that this theory is problematic as it is not conservative [2]. Let's add a new quantifier "Couple". And M means "Only a couple of x are y" and N represents "Only a couple of x are not y". If we use Peterson's rule, the table for Figure III should looks like the following table:

Table 2.13: *Figure III Affirmative + "Half" + "Couple"*

AAI	PAI	TAI	FAI	KAI	MAI	IAI
API	PPI	TPI	FPI	KPI	MPI	
ATI	PTI	TTI	FTI	KTI		
AFI	PFI	TFI	FFI			
AKI	PKI	TKI				
AMI	PMI					
AII						

TKI-III is considered to be valid if we use the rule of Peterson. But we have already shown that TKI-III is invalid when there is five quantifiers in Table 2. 8. So TKI-III will stay invalid in the extensions which means Peterson's rule is uncorrect when there is more than five quantifiers.

Take another example. Let's move out P "Almost-all" and B "Almost-all" from the basic Peterson's Intermediate syllogisms. Then we will get a four-quantifier Intermediate syllogisms and the Figure III(affirmative) table is

Table 2.14: *Figure III Affirmative - "Almost-all"*

AAI	TAI	KAI	IAI
ATI	TTI		
AKI			
AII			

We can see this table is not the form of triangle as there is no P anymore.

From the two examples we can conclude that the rule of Peterson is not right all the time.

2.3.2 Turunen's work

For Peterson's Intermediate syllogisms, Turunen uses MV-algebra to determine the validity.

First, he gives every statement an element q called *grade* such that $q \in (0, 1]$. The grade table is the following [2]:

Table 2.15: *Grade table*

Affirmative	Grade	Negative	Grade
A	1	**E**	0
P	p	**B**	1 - p
T	t	**D**	1 - t
K	k	**G**	1 - k
I	ε	**O**	1 - ε

In this table, we have several rules based on the contrary pairs:

- $0 < \varepsilon < 1 - p < k < \frac{1}{2} < t < p < 1$;
- $k + p > 1$;
- $t + k \leq 1$.

Second, Turunen gives the definition of MV-algebra and lattice. Here is the standard example of an MV-algebra where $x, y \in [0, 1]$.

- $x \oplus y = \min\{ x + y, 1\}$;
- $x^* = 1 - x$;
- $x \odot y = \max\{ x + y - 1, 0\}$;
- $x \vee y = \max\{ x, y\}$;

- $x \wedge y = \min\{\ x,\ y\}$;
- $x \to y = \min\{\ 1 - x + y,\ 1\}$.

Third, he gives several rules and theorems for the valid Peterson's Intermediate syllogisms based on some proofs.

For valid syllogisms in Figure I:

Affirmative case:

1. The major premise is A "All x are y";
2. Represent the grade of the miner premise by W and the grade of the conclusion as U, then the syllogism is valid if and only if $U^* \oplus W = 1$ or $U \leq W$.

We can use the rule to compute whether it is right or not. The following is the computing table for Figure I Affirmative computing case:

Table 2.16: *Figure I Affirmative computing case*

$U^* \oplus W$	(A, 1)	(P, p)	(T, t)	(K, k)	(I, ε)
$(A, 1)^* = 0$	1	p	t	k	ε
$(P, p)^* = 1 - p$	1	1	$1 - p + t$	$1 - p + k$	$1 - p + \varepsilon$
$(T, t)^* = 1 - t$	1	1	1	$1 - t + k$	$1 - t + \varepsilon$
$(K, k)^* = 1 - k$	1	1	1	1	$1 - k + \varepsilon$
$(I, \varepsilon)^* = 1 - \varepsilon$	1	1	1	1	1

From this table we can see that the results is the same as Peterson's conclusion in his book.

For negative case:

1. The major premise is the statement E "All x are not y";
2. Represent the grade of the miner premise by W and the grade of the conclusion as U, then the syllogism is valid if and only if $U \oplus W = 1$.

For valid syllogisms in Figure II:

For negative case II:

1. The major premise is the statement A "All x are y";
2. Represent the grade of the miner premise by W and the grade of the conclusion as U, then the syllogism is valid if and only if $W^* \oplus U = 1$.

For negative case II:

1. The major premise is the statement E "All x are not y";
2. Represent the grade of the miner premise by W and the grade of the conclusion as U, then the syllogism is valid if and only if $W \oplus U = 1$.

The following is the computing table for Figure II negative computing case II:

Table 2.17: *Figure II negative computing case II*

$U \oplus W$	(A, 1)	(P, p)	(T, t)	(K, k)	(I, ε)
(E, 0)	1	p	t	k	ε
(B, 1 - p)	1	1	1 - p + t	1 - p + k	1 - p + ε
(D, 1 - t)	1	1	1	1 - t + k	1 - t + ε
(G, 1 - k)	1	1	1	1	1 - k + ε
(O, 1 - ε)	1	1	1	1	1

For valid syllogisms in Figure III:

Affirmative case:

1. The conclusion is the statement I "Some x are y";
2. Represent the grade of the major premise by V and the grade of the miner premise as W, then the syllogism is valid if and only if $V \odot W \neq 0$.

We can see the following table for the computing of Figure III affirmative case:

For negative case:

1. The major premise is the statement O "Some x are not y";
2. Represent the grade of the major premise by V and the grade of the miner premise as W, then the syllogism is valid if and only if $V^* \odot W \neq 0$.

Table 2.18: Figure III Affirmative computing case

V $\odot$ W	(A, 1)	(P, p)	(T, t)	(K, k)	(I, ε)
(A, 1)	1	p	t	k	ε
(P, p)	p	2 * p - 1	$p + t$ - 1	$p + k$ - 1	0
(T, t)	t	$t + p$ - 1	2 * t - 1	0	0
(K, k)	k	$p + k$ - 1	0	0	0
(I, ε)	ε	0	0	0	0

For valid syllogisms in Figure IV:

In this case, all the valid syllogisms should fall between Aristotelian syllogisms when they are in Figure IV.

For example, AED-IV is valid because it is between AEE-IV and AEO-IV, ETO-IV is valid because it is between EAO-IV and EIO-IV.

For n-quantity valid syllogisms:

After using MV-algebra to study Peterson's Intermediate syllogisms, Turunen finds that the rules in Peterson's book are not well used when applying in other n-quantify syllogisms.

Based on the MV-algebra analysis on the Peterson's Intermediate syllogisms, Turunen gives an algorithm to produce valid intermediate syllogisms for n-quantity syllogisms [2].

At the end of Turunen's study, he gives an relationship between syllogisms and fuzzy logic which may be the further study.

2.4 Summary

At the beginning of this chapter, we talk about the history of Aristotelian syllogisms and give definitions of syllogisms. We also show the validity of syllogisms and how we can use Venn diagram to determine the validity of Aristotelian syllogisms.

In the second section, we introduce Peterson's Intermediate syllogisms based on the generalized quantifiers. We also give the conclusion that there are 105 valid syllogisms in this case. However, we denote that the rule of Peterson is not correct when applying in other n-quantify syllogisms.

After that, we talk about Turunen's study in MV-algebra with intermediate syllogisms. He points out the relation between MV-algebra and the valid syllogisms. Turunen also gives an algorithm for n-quantify syllogisms at the end of his article.

In chapter 3, we will give a percentage method to determine the validity of Intermediate syllogisms and also use solid mathematics to prove it.

3. PERCENTAGE METHOD

3.1 Overview

In chapter 2, we talk about the Peterson's Intermediate syllogisms and give several rules to determine the validity of intermediate syllogisms based on Peterson and Turunen's work. However, we point out that some rules maybe unavailable when talking about other n-quantifier Intermediate syllogisms.

In this chapter, we will present a new terminology called percentage method to determine the validity of syllogisms which could be Aristotelian syllogisms, Peterson's Intermediate syllogisms or other n-quantify syllogisms. Similar thought has been studied but not presented deeply [15, 16].

3.2 Mathematical foundation

We have already studied the Aristotelian syllogisms and give the form like the following one:

$$\begin{array}{c} \text{All M are P (Major premise)} \\ \underline{\text{Some S are not M (Miner premise)}} \\ \text{Some S are P (Conclusion)} \end{array}$$

Definition 2.1. *Related to the quantify in a statement, a degree means the percentage of x in or not in y. And we use* ***a*** *to denote the degree of Major premise,* ***b*** *to denote the degree of Miner premise and* ***c*** *to denote the degree of conclusion.*

Then, the example above can be represented as:

$$\begin{array}{c} \mathbf{a}\text{(percentage) M are P (Major premise)} \\ \underline{\mathbf{b}\text{(percentage) S are not M (Miner premise)}} \\ \mathbf{c}\text{(percentage) S are P (Conclusion)} \end{array}$$

From the definition, we can easily see the degree of "All" is 100% while the degree of "Some" should be smaller than 100% but bigger than 0%. In fact, the degree of "Some" should be very small and we could assign 1% or 0.1% to this quantify. Other quantifiers in Intermediate syllogisms should depend on the actual situation.

For statements "All x are y" and "All x are not y", they have the same degree. Which means we do not need to consider the affirmative or negative when we are talking about the degree. So I "Some x are y" and O "Some x are not y" has the same degree.

As there are four Figures we talk about in chapter 2, and every Figure has affirmative or negative case based on the conclusion. So we will talk about 8 cases in total and every section a Figure.

If we also consider the affirmative or negative of Major premise and Miner premise, then every case may have 4 subcases as the following:

Subcase 1

Affirmative (Major premise)
Affirmative (Miner premise)
Affirmative/Negative (Conclusion)

Subcase 2

Affirmative (Major premise)
Negative (Miner premise)
Affirmative/Negative (Conclusion)

Subcase 3

Negative (Major premise)
Affirmative (Miner premise)
Affirmative/Negative (Conclusion)

Subcase 4

Negative (Major premise)
Negative (Miner premise)
Affirmative/Negative (Conclusion)

3.3 Figure I

In this section, we will give two theorems about valid Intermediate syllogisms in Figure I. And here are the form of Figure I:

Figure I

$$\frac{\begin{array}{c}\text{A quantity } Q_1 \text{ of M are P (Short as MP)} \\ \text{A quantity } Q_2 \text{ of S are M (Short as SM)}\end{array}}{\text{A quantity } Q_3 \text{ of S are P}}$$

3.3.1 Figure I Affirmative case

Theorem 2.1. *For Figure I Affirmative case which means the conclusion is affirmative, the syllogism is valid if and only if $a = 100\%$, $b \geq c$, Major premise and Miner premise are both affirmative. Where a, b, c denote the degree of the quantifiers in Major premise, Miner premise and Conclusion.*

Proof. Here we give a Venn diagram to help us analysis.

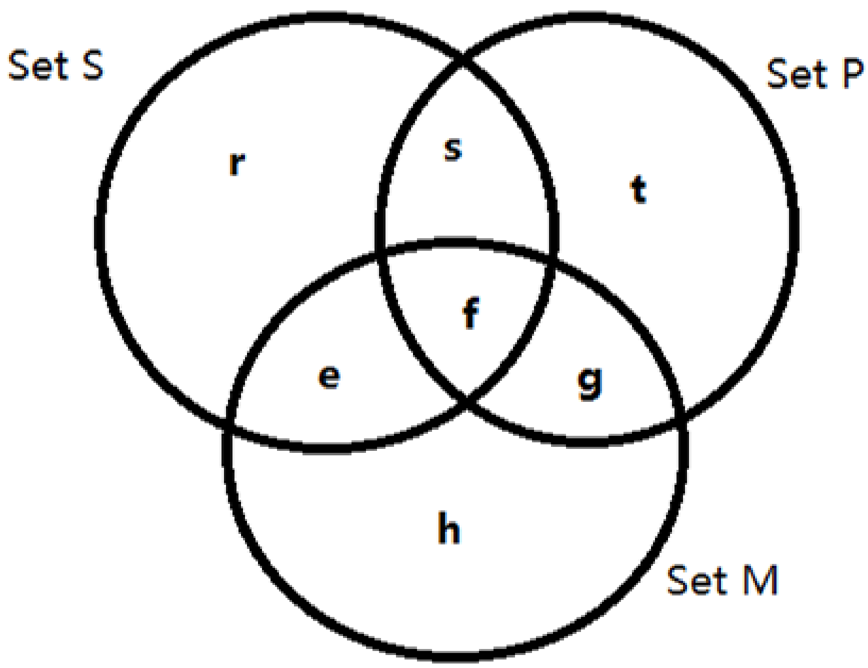

Figure 3.1: *Venn diagram for analysis*

Let's first consider subcase 1,

$$\frac{\begin{array}{c}\textbf{a}\text{(percentage) M are P}\\ \textbf{b}\text{(percentage) S are M}\end{array}}{\textbf{c}\text{(percentage) S are P}}$$

If a $<$ 100%, which means there maybe some M are not in P. No matter how much b of S in M, we do not know whether there is S in P as there is a possibility that e > 0, s + f = 0.

So we can conclude a should be 1, "All" M are P. Then we can see the amount of S in M are also in P. From the premise "All M are P" and "b S are M", we can conclude "c S are P" if b $\geq$ c.

So in Figure I Affirmative subcase 1, the syllogism is valid if and only if a = 100%, b $\geq$ c.

For subcase 2,

$$\frac{\begin{array}{c}\textbf{a}\text{(percentage) M are P}\\ \textbf{b}\text{(percentage) S are not M}\end{array}}{\textbf{c}\text{(percentage) S are P}}$$

Without considering the value of a, b, c, just let the s = f = 0 in the Venn diagram. Even the Major premise and Miner premise is true, the conclusion is negative. So there is no valid syllogisms in Figure I Affirmative subcase 2.

For subcase 3,

$$\frac{\begin{array}{c}\textbf{a}\text{(percentage) M are not P}\\ \textbf{b}\text{(percentage) S are M}\end{array}}{\textbf{c}\text{(percentage) S are P}}$$

the reason is the same as subcase 2 and there is no valid syllogisms.

For subcase 4,

$$\frac{\begin{array}{c}\textbf{a}\text{(percentage) M are not P}\\ \textbf{b}\text{(percentage) S are not M}\end{array}}{\textbf{c}\text{(percentage) S are P}}$$

as Major premise and Miner premise are negative, maybe all the three sets, S, M and P have no intersection. So no valid syllogisms in this subcase.

All in all, for Figure I Affirmative case, the syllogism is valid if and only if a = 100%, b $\geq$ c in subcase 1 where the both premises are affirmative.

Let's use Peterson's Intermediate syllogisms to test the theorem. To form valid syllogisms in Figure I and the conclusion is affirmative, then the following table is the result of the theorem(a = 100%, b $\geq$ c, Major premise and Miner premise are both affirmative):

Table 3.1: *Figure I Affirmative using percentage method*

AAA				
AAP	APP			
AAT	APT	ATT		
AAK	APK	ATK	AKK	
AAI	API	ATI	AKI	AII

The result is the same with Peterson's book and Turunen's work.

3.3.2 Figure I Negative case

Theorem 2.2. *For Figure I Negative case which means the conclusion is negative, the syllogism is valid if and only if a = 100%, b $\geq$ c, Major premise is negative and Miner premise is affirmative. Where a, b, c denote the degree of the quantifiers in Major premise, Miner premise and Conclusion.*

Proof.

First, we consider subcase 1,

$$\begin{array}{c}\textbf{a}\text{(percentage) M are P}\\ \underline{\textbf{b}\text{(percentage) S are M}}\\ \textbf{c}\text{(percentage) S are not P}\end{array}$$

As both Major premise and Miner premise are affirmative, there is a possibility that S, P and M are the same, then the conclusion is affirmative. So in this subcase, we have no valid syllogisms.

For subcase 2,

$$\frac{\begin{array}{c}\textbf{a}\text{(percentage) M are P}\\ \textbf{b}\text{(percentage) S are not M}\end{array}}{\textbf{c}\text{(percentage) S are not P}}$$

As the Major premise is affirmative, then f + g > 0, the Miner premise is negative, then r + s > 0. From the two premises we can not conclude that some percentage of s are not p, because it is ok if r = e = 0 and s > 0 which means all S are P. So in this case, there is no valid syllogisms.

For subcase 3,

$$\frac{\begin{array}{c}\textbf{a}\text{(percentage) M are not P}\\ \textbf{b}\text{(percentage) S are M}\end{array}}{\textbf{c}\text{(percentage) S are not P}}$$

If a < 100%, which means there is some M are in P. No matter how much b of S in M, we can not conclude that "some S are not P". Because there is a possibility that S and P are the same but different with M.

So a should be 100%, which means "All" M are not P. Then we can see the amount of S in M are not in P. From the premise "All M are not P" and "b S are M", we can conclude "c S are not P" if b $\geq$ c.

So in subcase 3, the syllogism is valid if and only if a = 100%, b $\geq$ c.

For subcase 4,

$$\frac{\begin{array}{c}\textbf{a}\text{(percentage) M are not P}\\ \textbf{b}\text{(percentage) S are not M}\end{array}}{\textbf{c}\text{(percentage) S are not P}}$$

Just assume that S and P are the same set, then no matter what the value of a and b is, the conclusion is affirmative. So in this subcase, there is no valid syllogisms.

All in all, for Figure I Negative case, the syllogism is valid if and only if a = 100%, b $\geq$ c in subcase 3 where the Major premise is negative statement and Miner premise is affirmative.

We can also use the theorem to compute the valid Peterson's Intermediate syllogisms in Figure I Negative case.

The computing result is the following, here we have a = 100%, b $\geq$ c, Major premise is negative and Miner premise is affirmative.

Table 3.2: *Figure I Negative using percentage method*

EAE				
EAB	EPB			
EAD	EPD	ETD		
EAG	EPG	ETG	EKG	
EAO	EPO	ETO	EKO	EIO

3.4 Figure II

In this section, we will give a theorem about valid Intermediate syllogisms in Figure II. And here are the form of Figure II:

$$\begin{array}{c}\textbf{Figure II}\\ \text{A quantity } Q_1 \text{ of P are M (Short as PM)}\\ \underline{\text{A quantity } Q_2 \text{ of S are M (Short as SM)}}\\ \text{A quantity } Q_3 \text{ of S are P}\end{array}$$

3.4.1 Figure II Affirmative case

Theorem 2.3. *For Figure II Affirmative case which means the conclusion is affirmative, There is no valid syllogisms.*

Proof.

For subcase 1,

$$\begin{array}{c}\textbf{a}\text{(percentage) P are M}\\ \underline{\textbf{b}\text{(percentage) S are M}}\\ \textbf{c}\text{(percentage) S are P}\end{array}$$

For this case, we can see that no matter how the degree of Major premise and Miner premise is, as they are all affirmative, it is possible that S and P may have different part of M which means S and P have no intersection. The figure below can easily prove this. So there is no valid syllogisms in this subcase.

For subcase 2,

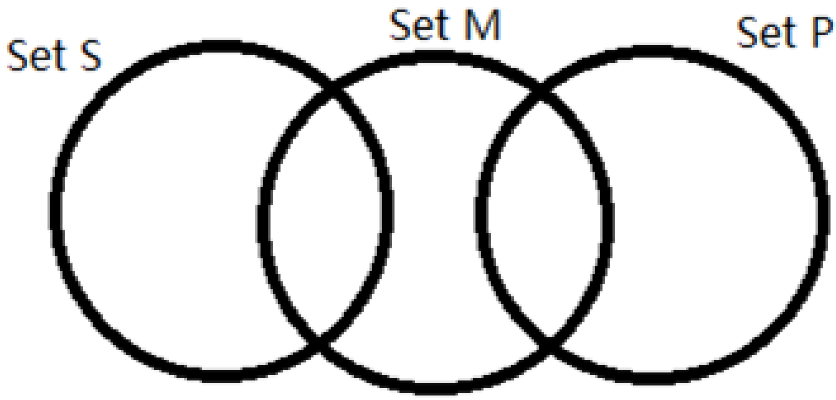

Figure 3.2: *Venn diagram for analysis 1*

$$\frac{\begin{array}{c}\textbf{a}\text{(percentage) P are M}\\ \textbf{b}\text{(percentage) S are not M}\end{array}}{\textbf{c}\text{(percentage) S are P}}$$

subcase 3,

$$\frac{\begin{array}{c}\textbf{a}\text{(percentage) P are not M}\\ \textbf{b}\text{(percentage) S are M}\end{array}}{\textbf{c}\text{(percentage) S are P}}$$

and subcase 4,

$$\frac{\begin{array}{c}\textbf{a}\text{(percentage) P are not M}\\ \textbf{b}\text{(percentage) S are not M}\end{array}}{\textbf{c}\text{(percentage) S are P}}$$

We can also find from the Venn diagram above that every quantify could be in the premise, but the conclusion could still be negative.

So in Figure II, there is no valid syllogisms when the statement of conclusion is affirmative.

3.4.2 Figure II Negative case

Theorem 2.4. *For Figure II Negative case which means the conclusion is negative, the syllogism is valid if and only if* $a = 100\%$, $b \geq c$, *one of the Major premise and Miner premise is affirmative while the other is negative. Where a, b, c denote the degree of the quantifiers in Major premise, Miner premise and Conclusion.*

Proof.

First for subcase 1,

$$\frac{\begin{array}{c}\textbf{a}\text{(percentage) P are M}\\ \textbf{b}\text{(percentage) S are M}\end{array}}{\textbf{c}\text{(percentage) S are not P}}$$

No matter what the value of the degree is, there is a possibility that S and P have some intersection and also have intersection with M which means the conclusion could be affirmative. So there is no valid syllogism in this subcase.

For subcase 2,

$$\frac{\begin{array}{c}\textbf{a}\text{(percentage) P are M}\\ \textbf{b}\text{(percentage) S are not M}\end{array}}{\textbf{c}\text{(percentage) S are not P}}$$

If $a < 100\%$, which means "Some P are not M". Then whatever the value of b is, S could be all in the P part where there is no M. So if we want to get a negative conclusion, the value of a should be 1, "All P are M". Then b of S are not M can result at least b of S are not P.

So in this subcase, to make the syllogisms valid, a should be 100% and $b \geq c$.

For subcase 3,

$$\frac{\begin{array}{c}\textbf{a}\text{(percentage) P are not M}\\ \textbf{b}\text{(percentage) S are M}\end{array}}{\textbf{c}\text{(percentage) S are not P}}$$

If $a < 100\%$, which means "Some P are M". Then whatever the value of b is, S could be all in the P part where there is M. So if we want to get a negative conclusion, the value of a should be 100%, "All P are not M". Then b of S are M can result at least b of S are not P.

So in this subcase, to make the syllogisms valid, a should be 100% and b $\geq$ c.

For subcase 4,

$$\begin{array}{c} \textbf{a}\text{(percentage) P are not M} \\ \underline{\textbf{b}\text{(percentage) S are not M}} \\ \textbf{c}\text{(percentage) S are not P} \end{array}$$

It is possible that S and P are the same, but at least some of them are not M. So in this subcase, there is no valid syllogisms.

All in all, for Figure II Negative case, the syllogism is valid if and only if a = 100%, b $\geq$ c in subcase 2 or subcase 3.

We can also use the theorem to compute the valid Peterson's Intermediate syllogisms in Figure II Negative case. The computing results are the following, here we have a = 100%, b $\geq$ c, subcase 2 or subcase 3.

Table 3.3: *Figure II Negative subcase 2*

AEE				
AEB	ABB			
AED	ABD	ADD		
AEG	ABG	ADG	AGG	
AEO	ABO	ADO	AGO	AOO

Table 3.4: *Figure II Negative subcase 3*

EAE				
EAB	EPB			
EAD	EPD	ETD		
EAG	EPG	ETG	EKG	
EAO	EPO	ETO	EKO	EIO

The results are the same as Peterson's book.

3.5 Figure III

In this section, we will give theorems about valid intermediate syllogisms in Figure III. And here are the form of Figure III:

$$\begin{array}{c}\textbf{Figure III}\\ \text{A quantity } Q_1 \text{ of M are P(Short as MP)}\\ \underline{\text{A quantity } Q_2 \text{ of M are S (Short as MS)}}\\ \text{A quantity } Q_3 \text{ of S are P}\end{array}$$

3.5.1 Figure III Affirmative case

Theorem 2.5. *For Figure III Affirmative case which means the conclusion is affirmative, the syllogism is valid if and only if a + b > 100%, c should be the smallest degree of quantify(the related quantify is "Some"), both of the Major premise and Miner premise are affirmative. Where a, b, c denote the degree of the quantifiers in Major premise, Miner premise and Conclusion.*

Proof. First for subcase 1,

$$\begin{array}{c}\mathbf{a}\text{(percentage) M are P}\\ \underline{\mathbf{b}\text{(percentage) M are S}}\\ \mathbf{c}\text{(percentage) S are P}\end{array}$$

If a + b < 100% (or a + b = 100%), which means that maybe a possibility that S and P have no intersection. Then the conclusion could be negative.

But if a + b > 100%, which means it is impossible that some M not in P and not in S. In other words, some M are both in S and P. However, we do not know the whole amount of S, so the quantify in the conclusion should be only "Some" which is the smallest of quantifiers.

For subcase 2,

$$\begin{array}{c}\mathbf{a}\text{(percentage) M are P}\\ \underline{\mathbf{b}\text{(percentage) M are not S}}\\ \mathbf{c}\text{(percentage) S are P}\end{array}$$

No matter how much M are in P, it is possible that M and S have no intersection, then we can not conclude that some S are P. It is possible that M and P are the same but totally different with S. Then the conclusion could be negative. So there are no valid syllogisms in this subcase.

For subcase 3,

$$\frac{\begin{array}{c}\mathbf{a}(\text{percentage}) \text{ M are not P} \\ \mathbf{b}(\text{percentage}) \text{ M are S}\end{array}}{\mathbf{c}(\text{percentage}) \text{ S are P}}$$

The reason is the same as in subcase 2, there are no valid syllogisms in this subcase.

For subcase 4,

$$\frac{\begin{array}{c}\mathbf{a}(\text{percentage}) \text{ M are not P} \\ \mathbf{b}(\text{percentage}) \text{ M are not S}\end{array}}{\mathbf{c}(\text{percentage}) \text{ S are P}}$$

It is possible that M, P, S are three totally different set, which means no S are P. So no valid syllogisms in this subcase.

Overall, for Figure III Affirmative case, the syllogism is valid if and only if a + b $>$ 100%, c is the degree of "Some" and both of the premises are affirmative.

Using the theorem above, we have the following table for Figure III Affirmative case.

Table 3.5: *Figure III Affirmative subcase 1*

AAI	PAI	TAI	KAI	IAI
API	PPI	TPI	KPI	
ATI	PTI	TTI		
AKI	PKI			
AII				

3.5.2 Figure III Negative case

Theorem 2.6. *For Figure III Negative case which means the conclusion is negative, the syllogism is valid if and only if a + b $>$ 100%, c should be the smallest(the* *related quantify is "Some"), the Major premise is negative, and Miner premise is affirmative. Where a, b, c denote the degree of the quantifiers in Major premise, Miner premise and Conclusion.*

Proof. First for subcase 1,

$$\frac{\begin{array}{c}\mathbf{a}(\text{percentage) M are P}\\ \mathbf{b}(\text{percentage) M are S}\end{array}}{\mathbf{c}(\text{percentage) S are not P}}$$

In this subcase, there is a possibility that M, P and S are the same no matter what the value of a, b are. Then the conclusion could be affirmative. So no valid syllogisms here as the conclusion could be affirmative.

For subcase 2,

$$\frac{\begin{array}{c}\mathbf{a}(\text{percentage) M are P}\\ \mathbf{b}(\text{percentage) M are not S}\end{array}}{\mathbf{c}(\text{percentage) S are not P}}$$

Just assume that all M are in P and the rest of P is S, S is all in P. So there is no valid syllogisms in subcase 2.

For subcase 3,

$$\frac{\begin{array}{c}\mathbf{a}(\text{percentage) M are not P}\\ \mathbf{b}(\text{percentage) M are S}\end{array}}{\mathbf{c}(\text{percentage) S are not P}}$$

In this subcase, just as the reason in the subcase 1 of Figure III affirmative, which means a + b > 100%, c should be the smallest(the related quantify is "Some").

For subcase 4,

$$\frac{\begin{array}{c}\mathbf{a}(\text{percentage) M are not P}\\ \mathbf{b}(\text{percentage) M are not S}\end{array}}{\mathbf{c}(\text{percentage) S are not P}}$$

It is possible that P and S are the same but M has no intersection with them. So no valid syllogisms in this subcase.

All in all, for Figure III Negative case, the syllogism is valid if and only if a + b > 100%, c is the smallest(the related quantify is "Some") in subcase 3.

Using the theorem above, we have the following table for Figure III Negative case.

Table 3.6: *Figure III Negative subcase 3*

EAO	BAO	DAO	GAO	OAO
EPO	BPO	DPO	GPO	
ETO	BTO	DTO		
EKO	BKO			
EIO				

3.6 Figure IV

In this section, we will give several theorems about valid intermediate syllogisms in Figure IV. And here are the form of Figure IV:

$$\begin{array}{c} \textbf{Figure IV} \\ \text{A quantity } Q_1 \text{ of P are M (Short as PM)} \\ \underline{\text{A quantity } Q_2 \text{ of M are S (Short as MS)}} \\ \text{A quantity } Q_3 \text{ of S are P} \end{array}$$

3.6.1 Figure IV Affirmative case

Theorem 2.7. *For Figure IV Affirmative case which means the conclusion is affirmative, the syllogism is valid if and only if b = 100%, c is the smallest(the related quantify is "Some"), both of the Major premise and Miner premise are affirmative. Where a, b, c denote the degree of the quantifiers in Major premise, Miner premise and Conclusion.*

Proof. First for subcase 1,

$$\begin{array}{c} \textbf{a}\text{(percentage) P are M} \\ \underline{\textbf{b}\text{(percentage) M are S}} \\ \textbf{c}\text{(percentage) S are P} \end{array}$$

In this subcase, some P is in M. But if not all M are S, then we can not conclude some S are in P. So b = 100%. And we do not know how much amount of S are P, so we choose the smallest one "Some". The syllogisms here are valid if and only if b = 100% and c is the degree of "Some".

For subcase 2,

$$\frac{\begin{array}{c}\mathbf{a}\text{(percentage) P are M}\\ \mathbf{b}\text{(percentage) M are not S}\end{array}}{\mathbf{c}\text{(percentage) S are P}}$$

It is possible that P and M are the same, but M and S have no intersection. Then the conclusion is negative. So no valid syllogisms in subcase 2.

For subcase 3,

$$\frac{\begin{array}{c}\mathbf{a}\text{(percentage) P are not M}\\ \mathbf{b}\text{(percentage) M are S}\end{array}}{\mathbf{c}\text{(percentage) S are P}}$$

It is possible that S and M are the same, but M and P have no intersection. Then the conclusion is negative. So no valid syllogisms in subcase 3.

For subcase 4,

$$\frac{\begin{array}{c}\mathbf{a}\text{(percentage) P are not M}\\ \mathbf{b}\text{(percentage) M are not S}\end{array}}{\mathbf{c}\text{(percentage) S are P}}$$

It is possible that P, S and M are totally different, then the conclusion is negative. So no valid syllogisms in subcase 4.

Overall, for Figure IV Affirmative case, the syllogism is valid if and only if b = 100%, c is the smallest(the related quantify is "Some") in subcase 1.

Using the theorem above, we have the following table for Figure IV Affirmative case.

Table 3.7: *Figure IV Affirmative subcase 1*

AAI	PAI	TAI	KAI	IAI

3.6.2 Figure IV Negative case

Theorem 2.8. *For Figure IV Negative case which means the conclusion is negative, the syllogism is valid if and only if a=100%, b = 100%, the Major premise is affirmative and Miner premise is negative; Or a = 100%, c is the degree of "Some", the Major premise is negative and Miner premise is affirmative. Where a, b, c denote the degree of the quantifiers in Major premise, Miner premise and Conclusion.*

Proof. First for subcase 1,

$$\frac{\begin{array}{c}\mathbf{a}\text{(percentage) P are M}\\ \mathbf{b}\text{(percentage) M are S}\end{array}}{\mathbf{c}\text{(percentage) S are not P}}$$

It is possible that P, M and S are the same, then the conclusion should be affirmative. So there is no valid syllogisms in this subcase.

For subcase 2,

$$\frac{\begin{array}{c}\mathbf{a}\text{(percentage) P are M}\\ \mathbf{b}\text{(percentage) M are not S}\end{array}}{\mathbf{c}\text{(percentage) S are not P}}$$

In this case, if not all P are M, then it is possible that S is the same as P or all S are in P. So a should be 100%. And if some M are S, it is possible that S and P are all in M and S and P are the same. So b should be 100%. In that case, c could be the degree of any quantifiers.

For subcase 3,

$$\frac{\begin{array}{c}\mathbf{a}\text{(percentage) P are not M}\\ \mathbf{b}\text{(percentage) M are S}\end{array}}{\mathbf{c}\text{(percentage) S are not P}}$$

In this case, if some P are M and some P are not M, then it is possible that all the P and M are in S. So a = 100%. If "All P are not M", then b of M are s can conclude some S are not P. However, we do not know b of M is how much of S, so c is the degree of "Some".

For subcase 4,

$$\frac{\begin{array}{c}\textbf{a}\text{(percentage) P are not M}\\ \textbf{b}\text{(percentage) M are not S}\end{array}}{\textbf{c}\text{(percentage) S are not P}}$$

It is possible that S and P are the same but have no intersection with M. So in this subcase, there is no valid syllogism.

All in all, for Figure IV Negative case, the syllogism is valid if and only if a=100%, b = 100% in subcase 2; Or a = 100%, c is the degree of "Some" in subcase 3.

The following are the tables calculated by using the theorem for Figure IV Negative case.

Table 3.8: *Figure IV Negative subcase 2*

AEE	AEB	AED	AEG	AEO

Table 3.9: *Figure IV Negative subcase 3*

EAO	EPO	ETO	EKO	EIO

3.7 Summary

At the beginning of this chapter, we define the degree of the quantify which means percentage in syllogisms without considering whether the statement is affirmative or negative. Simply speaking, "Almost-all x are y" and "Almost-all x are not y" have the same degree. Later, we present several theorems to determine the validity of syllogisms. The syllogisms can be Aristotelian syllogisms, Peterson's Intermediate syllogisms or other n-quantify syllogisms. From the proof of these theorems, we can easily find some rules about the valid syllogisms:

- at least one of the premises is affirmative;

- If the two premises are affirmative, then the conclusion is affirmative;
- If one premise ia negative, then the conclusion is negative;
- If any premise is non-universal, then the conclusion must have a quantity that is less than or equal to that premise.

As the theorems are simple but maybe a little complicated, here are several three-line form to determine the valid syllogisms.

Figure I

$$\frac{\begin{array}{c}\text{Affirmative, } a = 100\% \\ \text{Affirmative, } b \geq c\end{array}}{\text{Affirmative, } c > 0\%}$$

Figure I

$$\frac{\begin{array}{c}\text{Negative, } a = 100\% \\ \text{Affirmative, } b \geq c\end{array}}{\text{Negative, } c > 0\%}$$

Figure II

$$\frac{\begin{array}{c}\text{Affirmative, } a = 100\% \\ \text{Negative, } b \geq c\end{array}}{\text{Negative, } c > 0\%}$$

Figure II

$$\frac{\begin{array}{c}\text{Negative, } a = 100\% \\ \text{Affirmative, } b \geq c\end{array}}{\text{Negative, } c > 0\%}$$

Figure III

$$\frac{\begin{array}{c}\text{Affirmative, } a \\ \text{Affirmative, } a + b > 100\%\end{array}}{\text{Affirmative, } c \text{ is the degree of "Some"}}$$

Figure III

$$\frac{\begin{array}{c}\text{Negative, } a \\ \text{Affirmative, } a + b > 100\%\end{array}}{\text{Negative, } c \text{ is the degree of "Some"}}$$

Figure IV

Affirmative, a $>$ 0%

Affirmative, b $=$ 100%

Affirmative, c is the degree of "Some"

Figure IV

Affirmative, a $=$ 100%

Negative, b $=$ 100%

Negative, c $>$ 0%

Figure IV

Negative, a $=$ 100%

Affirmative, b $>$ 0%

Negative, c is the degree of "Some"

4. EXAMPLES

In chapter 3, we introduce percentage method with several theorems to determine the validity of Intermediate syllogisms. And this method is well used in Peterson's Intermediate syllogisms.

In this chapter, we will give both theoretical example and practical example to test our method.

4.1 Theoretical example

Peterson's Intermediate syllogisms have five quantifiers, and we give a 7-quantify intermediate syllogisms based on the Peterson's Intermediate syllogisms.

Here, we give other two quantifiers: "Half" and "Small part". Then we could have four statements: "Half x are y", "Half x are not y", "Small part of x are y" and "Small part of x are not y". The following is the table about the degree of the 7 quantifiers.

Table 4.1: *Degree of quantifiers in theoretical example*

Affirmative statements	Negative statements	Degree
A: All x are y	**E**: All x are not y	100%
P: Almost-all x are y	**B**: Almost-all x are not y	90%
T: Most x are y	**D**: Most x are not y	60%
R: Half x are y	**S**: Half x are not y	50%
K: Many x are y	**G**: Many x are not y	40%
W: Small part of x are y	**M**: Small part of x are not y	20%
I: Some x are y	**O**: Some x are not y	1%

From the theorems in chapter 3, we could determine the validity of the 7-quantify intermediate syllogisms. The result is the following tables. We can easily see that the table may not be a triangle especially in Figure III.

Table 4.2: *Figure I Affirmative of theoretical example*

AAA						
AAP	APP					
AAT	APT	ATT				
AAR	APR	ATR	ARR			
AAK	APK	ATK	ARK	AKK		
AAW	APW	ATW	ARW	AKW	AWW	
AAI	API	ATI	ARI	AKI	AWI	AII

Table 4.3: *Figure I Negative of theoretical example*

EAE						
EAB	EPB					
EAD	EPD	ETD				
EAS	EPS	ETS	ERS			
EAG	EPG	ETG	ERG	EKG		
EAM	EPM	ETM	ERM	EKM	EWM	
EAO	EPO	ETO	ERO	EKO	EWO	EIO

Table 4.4: *Figure II Negative case 1 of theoretical example*

AEE						
AEB	ABB					
AED	ABD	ADD				
AES	ABS	ADS	ASS			
AEG	ABG	ADG	ASG	AGG		
AEM	ABM	ADM	ASM	AGM	AMM	
AEO	ABO	ADO	ASO	AGO	AMO	AOO

Table 4.5: *Figure II Negative case 2 of theoretical example*

EAE						
EAB	EPB					
EAD	EPD	ETD				
EAS	EPS	ETS	ERS			
EAG	EPG	ETG	ERG	EKG		
EAM	EPM	ETM	ERM	EKM	EWM	
EAO	EPO	ETO	ERO	EKO	EWO	EOO

Table 4.6: *Figure III Affirmative case of theoretical example*

AAI	PAI	TAI	RAI	KAI	WAI	IAI
API	PPI	TPI	RPI	KPI	WPI	
ATI	PTI	TTI	RTI			
ARI	PRI	TRI				
AKI	PKI					
AWI	PWI					
AII						

Table 4.7: *Figure III Negative case of theoretical example*

EAO	BAO	DAO	SAO	GAO	MAO	OAO
EPO	BPO	DPO	SPO	GPO	MPO	
ETO	BTO	DTO	STO			
ERO	BRO	DRO				
EKO	BKO					
EWO	BWO					
EIO						

Table 4.8: *Figure IV of theoretical example*

Affirmative case	Negative case 1	Negative case 2
AAI	AEE	EAO
PAI	AEB	EPO
TAI	AED	ETO
RAI	AES	ERO
KAI	AEG	EKO
WAI	AEM	EWO
IAI	AEO	EIO

4.2 Practical example

Here, we give a practical example. We all know the difference between European Union, Eurozone and Schengen Area. Let's represent European Union as EU, Eurozone as EZ and Schengen Area as SA. Then we have the following sets:

EU = {Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, United Kingdom}

EZ = {Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Portugal, Slovakia, Slovenia, Spain}

SA = {Austria, Belgium, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Italy, Latvia, Liechtenstein, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden, Switzerland}.

The following is the Venn diagram for this practical example:

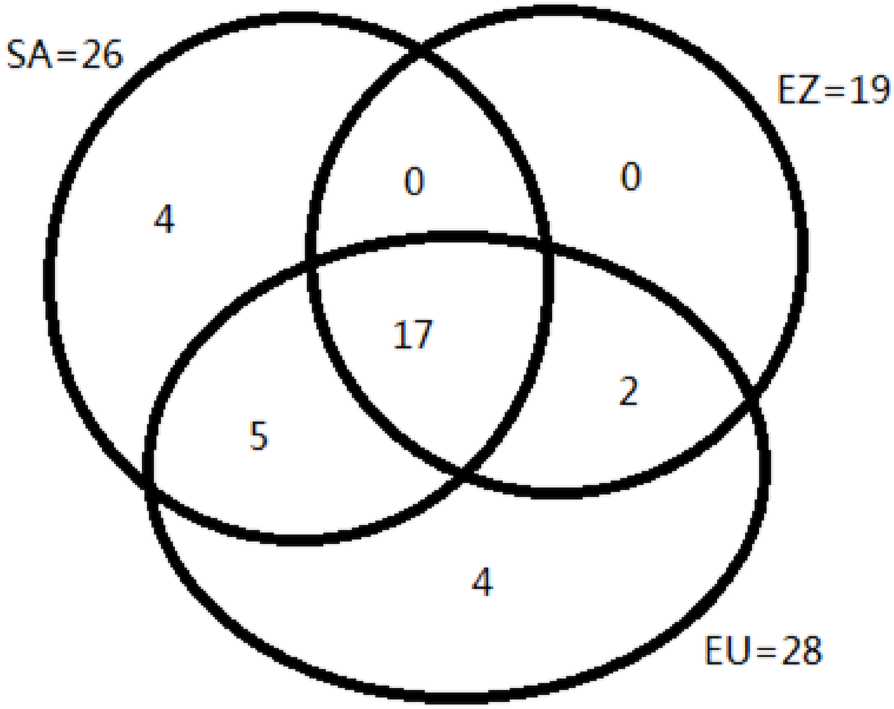

Figure 4.1: *Venn diagram for practical example*

Here, we give four quantifiers: "All", "Almost-all", "Many" and "Some". Then we could have eight statements. And every quantifies could have different degree. The following is the table of degree for these quantifiers.

Table 4.9: *Degree of quantifiers in practical example*

Affirmative statements	Negative statements	Degree
A: All x are y	**E**: All x are not y	100%
P: Almost-all x are y	**B**: Almost-all x are not y	80%
K: Many x are y	**G**: Many x are not y	30%
I: Some x are y	**O**: Some x are not y	10%

From the Venn diagram and the degree, we have many statements such as "All EZ are EU", "Many SA are EZ", "Almost-all EZ are SA" and so on. Using these statements, we can easily deduce other statements.

For example, if we have statements "All EZ are EU" and "Many SA are EZ", we could have several conclusions.

There are two "EZ" in the two premises, so the conclusion is about the relationship between EU and SA. We can easily see this should belongs to Figure IV Affirmative case or Figure I Affirmative case. From the results in chapter 3, we have

Figure IV

Affirmative, a $>$ 0%

<u>Affirmative, b $=$ 100%</u>

Affirmative, c is the degree of "Some"

We can let "Many SA are EZ" be the Major premise and "All EZ are EU" be the Miner premise, then we have the following valid syllogism:

Figure IV

Many SA are EZ

<u>All EZ are EU</u>

Some EU are SA

From the Venn diagram above, we can also see the conclusion "Some EU are SA" is right.

For Figure I, we have

Figure I

Affirmative, a $=$ 100%

<u>Affirmative, b $\geq$ c</u>

Affirmative, c $>$ 0%

Let "All EZ are EU" be the Major premise and "Many SA are EZ" be the Miner premise, then we have the following valid syllogism:

Figure I

"All EZ are EU"
"Many SA are EZ"

Many SA are EU

and

Figure I

"All EZ are EU"
"Many SA are EZ"

Some SA are EU

We have two valid Figure I affirmative syllogisms because the degree of "Many" is bigger than "Some".

5. COMPARISON WITH OTHERS' METHODS

In Peterson's book, the author uses linguistics to analysis the generalized quantifiers and the intermediate syllogisms. And Peterson gives the valid syllogisms when there are five quantifiers: "All", "Some", "Almost-all", "Most" and "Many" which we called Peterson's Intermediate syllogisms. Peterson also defines a sophisticated extended Venn diagram method to find valid syllogisms.

Peterson also finds that these valid syllogisms have some triangle rules. It is quite common to assume that when there are more quantifiers or less quantifiers, the valid syllogisms could also be the triangle. Peterson gives such triangle rules without proof as it would be hard to use his extended Venn diagram method to analysis as the meaning of intermediate quantifiers would be more complex.

Turunen shows how MV-algebra are related to Intermediate syllogisms. In his article, every quantify could have two statements: the affirmative statement and the negative one. And Turunen gives every statement a different grade. For example, there are five quantifiers and ten statements in Peterson's Intermediate syllogisms. He give ten grade to these statements. And based on the relationship of these statements, the grade should also be strict to these relationship.

Also, Turunen gives the definition of algebra and introduce the standard structure of MV-algebra. Based on the theory of MV-algebra and observation of valid syllogisms in Peterson's Intermediate syllogisms, he gives some mathematical formulas to determine the valid ones. Some proofs are given to address his method.

Through analysis of some other intermediate syllogisms, Turunen finds the rules in Peterson's book are wrong sometimes. To fix this problem, he introduces an algorithm to determine the valid syllogisms when there are more quantifiers. The algorithm is well used, however, no proofs are given. At the end of his article, it is quite good to relate fuzzy logic with intermediate syllogisms.

Comparing Turunen's approach, the thesis does not talk about the Peterson's Intermediate syllogisms first but the intermediate syllogisms which could be 2-quantifier, 3-quantifier and even more quantifiers. Which means the Aristotelian syllogisms and

Peterson's Intermediate syllogisms are just special syllogisms. And the two kinds syllogisms can also use this method to determine the validity of syllogisms.

And we introduce the degree of quantifiers which is different with Turunen's grade as every quantify has a degree but not every statement.

In this thesis, we use the Venn diagram with a slightly different terminology to study Intermediate syllogisms and give some better results.

6. CONCLUSION

This thesis work presented a new terminology connected with Venn diagram called percentage method to determine the validity of Intermediate syllogisms. In addition, we give proofs and examples to test the method. Also, we compare the percentage method with others's work.

Chapter 1 started with the introduction of the motivation of this thesis work, then we list the organization of this thesis.

In Chapter 2, we talked about the history of Aristotelian syllogisms which have only two quantifiers and reviewed the valid ones. And we gave the definitions of generalized quantifiers and intermediate syllogisms. Generalized quantifiers are quantifiers between "All" and "Some". Then we introduce Peterson's Intermediate syllogisms which have five quantifiers: "All", "Some", "Almost-all", "Most" and "Many". We show the valid Peterson's Intermediate syllogisms by citing the rules and results in Peterson's book. We also talked about Turunen's work and gave definitions of MV-algebra and grade in his article. Advantages and disadvantages of these works are also talked.

In Chapter 3, we gave the percentage terminology to determine the validity of intermediate syllogisms which could have many quantifiers. First, we introduced the definition of degree which could be the meaning of percentage. And we also gave several subcases of every Figure. Then for every Figure, we talked about different situations based and gave the condition of the valid intermediate syllogisms. We also use Venn diagram and logic to prove the terminology. What's more, Aristotelian syllogisms and Peterson's Intermediate syllogisms are used to test the method as we already know the valid ones. At last, we gave several rules which are similar in Peterson's book.

In Chapter 4, some examples are gave to test the terminology we introduced in chapter 3. We first talked about the syllogisms with seven quantifiers and gave results. And later we gave practical examples to show how to use the results in our life.

In Chapter 5, we compared the percentage method with Peterson's work in his book and Turunen's algebra study in his article. Through comparison, we found this terminology is easy to understand and apply in real life situation.

However, a solid way to determine the validity of Intermediate syllogisms, when a set of m quantifiers is given, should always begin by a linguistic analysis of the relation of these quantifiers. A crucial question is to answer when "a quantify X of A are B" and "a quantify Y of A are not B" can be true. And this leads to the study of contradictory statements by Peterson and analysis by Turunen we omit here.

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